Bounded-Error Identification of Linear Systems with Input and Output Backlash

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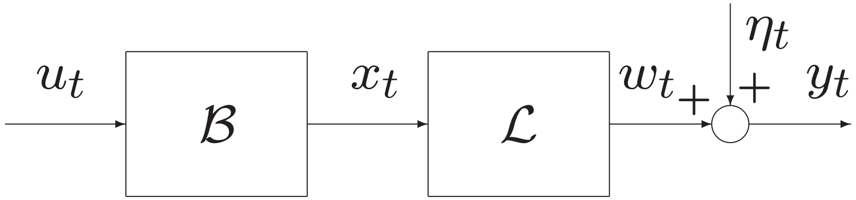
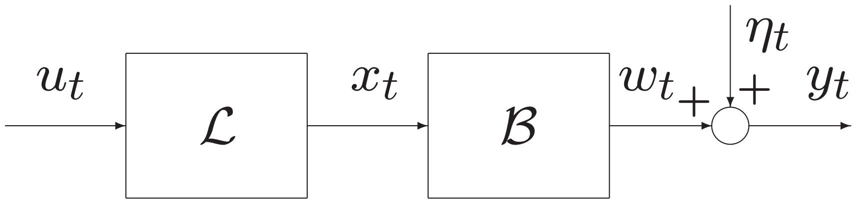
System Description

Linear dynamical system with input backlash



Linear dynamical system with output backlash



u_t: known input signal

y_t: noise-corrupted measurement of w_t

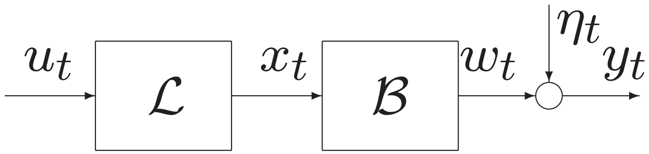
x_t: not measurable inner signal

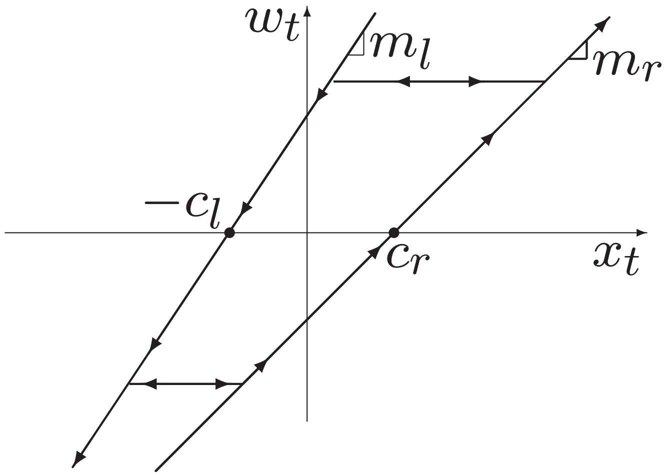
$\mathcal{B}$: backlash nonlinearity

$\mathcal{L}$: linear dynamic subsystem

$|\eta_t| \leq \Delta\eta_t$; $\Delta\eta_t$ known (Set-Membership characterization)

System Description





$$w_t = \mathcal{B}(x_t) = \begin{cases} m_l(x_t + c_l) & \text{for } x_t \leq \frac{w_{t-1}}{m_l} - c_l \\ m_r(x_t - c_r) & \text{for } x_t \geq \frac{w_{t-1}}{m_r} + c_r \\ w_{t-1} & \text{for } \frac{w_{t-1}}{m_l} - c_l < x_t < \frac{w_{t-1}}{m_r} + c_r \end{cases}$$

$$\mathcal{L} : x_t = -\sum_{i=1}^{na} a_i x_{t-i} + \sum_{j=0}^{nb} b_j u_{t-j}$$

Identification of linear systems with backlash

- **Aim of the work:** compute bounds on the backlash parameters $\gamma^\top = [m_l \; c_l \; m_r \; m_r]$ and linear block parameters $\theta^\top = [a_1 \ldots a_{na} \; b_0 \; b_1 \ldots b_{nb}]$.

- Parameter bound computation of linear systems with backlash is NP-hard in the size of the experimental data sequence

$$\Downarrow$$

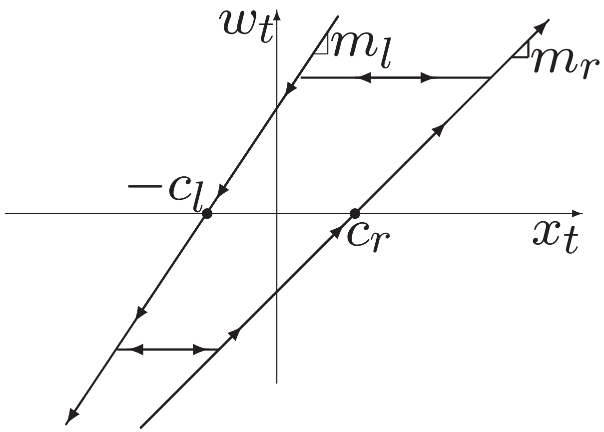
Computationally tractable relaxations are needed

Feasible parameter set (FPS)

In bounded-error (or set-membership) context, all the system parameters γ and θ consistent with the measurement data sequence, the assumed model structure and the error bounds are feasible solution to the identification problem (and are said to belong to the feasible parameter set $\mathcal{D}_{\gamma\theta}$).

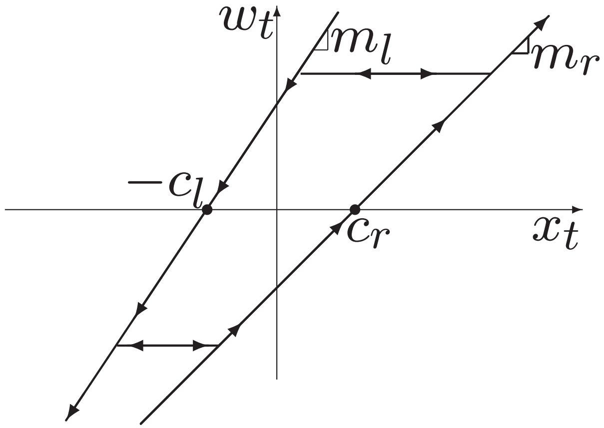
How to construct the Feasible Parameter Set?

Backlash nonlinearity



Can the backlash nonlinearity be inverted?

Backlash nonlinearity



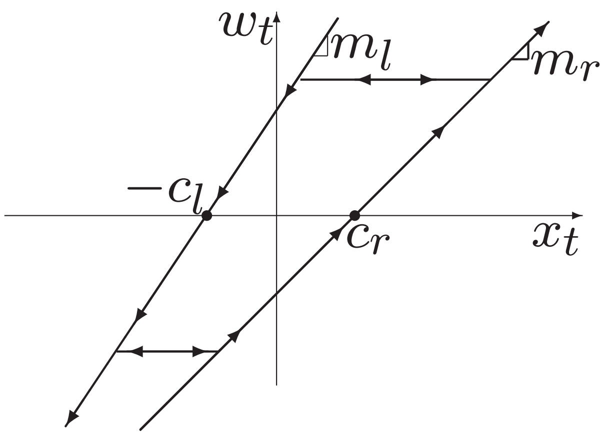
Definition 1: $\mathcal{Y}^r$ (*right-invertible output sequence*)

$$\mathcal{Y}^r = \{y_t \in \mathbb{R} : y_t - y_{t-1} > \Delta\eta_t + \Delta\eta_{t-1}\}$$

Definition 2: $\mathcal{Y}^l$ (*left-invertible output sequence*)

$$\mathcal{Y}^l = \{y_t \in \mathbb{R} : y_t - y_{t-1} < -\Delta\eta_t - \Delta\eta_{t-1}\}$$

Backlash nonlinearity



Proposition 1: If $y_t \in \mathcal{Y}^r \Rightarrow x_t = \frac{w_t}{m_r} + c_r$

Proposition 2: If $y_t \in \mathcal{Y}^l \Rightarrow x_t = \frac{w_t}{m_l} - c_l$

Proposition 3: If $y_t \in \mathcal{Y}^r \cup \mathcal{Y}^l \Rightarrow x_t = \left(\frac{w_t}{m_r} + c_r\right) \chi_{\mathcal{Y}^r}(y_t) + \left(\frac{w_t}{m_l} - c_l\right) \chi_{\mathcal{Y}^l}(y_t)$

$$\Rightarrow m_r m_l x_k = m_l \left(y_k - \eta_k + m_r c_r\right) \chi_{\mathcal{Y}^r}(y_k) + m_r \left(y_k - \eta_k - m_l c_l\right) \chi_{\mathcal{Y}^l}(y_k)$$

Feasible parameter set (FPS)

The FPS $\mathcal{D}_{\gamma\theta}$ is the projection on the parameter space of the set $\mathcal{D}$ of all system parameters γ-θ, noise samples and η_t and inner signals x_t consistent with the measurement data sequence, the assumed model structure and the error bounds, given by:

$$
\begin{aligned}
\mathcal{D} = \Big\{ (\gamma, \theta, x, \eta) : x_k &= -\sum_{i=1}^{na} a_i x_{k-i} + \sum_{j=1}^{nb} b_j u_{k-j}; \\
m_r m_l x_k &= m_l \left(y_k - \eta_k + m_r c_r \right) \chi_{\mathcal{Y}^r}(y_k) + m_r \left(y_k - \eta_k - m_l c_l \right) \chi_{\mathcal{Y}^l}(y_k); \\
|\eta_k| &\leq \Delta\eta_k, \quad k : y_k \in \mathcal{Y}^r \cup \mathcal{Y}^l \Big\}
\end{aligned}
$$

Computation of parameter bounds

- Exact parameter bounds:

$$\underline{\gamma}_k = \min_{(\gamma,\theta,x,\eta)\in\mathcal{D}} \gamma_k, \quad \overline{\gamma}_k = \max_{(\gamma,\theta,x,\eta)\in\mathcal{D}} \gamma_k$$

$$\underline{\theta}_j = \min_{(\gamma,\theta,x,\eta)\in\mathcal{D}} \theta_j, \quad \overline{\theta}_j = \max_{(\gamma,\theta,x,\eta)\in\mathcal{D}} \theta_j$$

- Parameter Uncertainty Intervals:

$$PUI_{\gamma_k} = \left[\underline{\gamma}_k;\ \overline{\gamma}_k\right] \quad PUI_{\theta_j} = \left[\underline{\theta}_j;\ \overline{\theta}_j\right]$$

Remark 1: The system parameters γ-θ, the inner signals x_t and the noise samples η_t are decision variables in the above optimization problem $\Rightarrow$ The number of optimization variables increases with the number of measurements

Remark 2: $\mathcal{D}$ is a nonconvex set described by polynomial constraints $\Rightarrow$ exact bound computation requires to solve a set of nonconvex optimization problems

Computation of parameter bounds

- Standard nonlinear optimization tools can not be exploited to compute bounds on γ_k (resp. θ_j) since they can trap in local minima

$$\Downarrow$$

The true value is not guaranteed to lie within the computed bounds

- Relax original identification problems to convex optimization problems

$$\Downarrow$$

Guaranteed (relaxed) bounds on each parameter γ_k (resp. θ_j) can be evaluated

Computation of relaxed PUI: LMI relaxation

- *General Idea*

  Exploit LMI relaxation for semialgebraic optimization problems

  SOS decomposition (**G. Chesi et al. (1999), P. Parrillo (2003)**)

  Theory of moments (**J. B. Lasserre (2001)**)

- *Computational complexity*

  Due to the large number of optimization variables and constraints involved in the identification problems, such LMI relaxation techniques leads, in general, to untractable SDP problems

The peculiar structured sparsity of the formulated identification problems can be used to reduce the computational complexity of such LMI-relaxation techniques in computing parameter bounds

Computation of relaxed bounds: exploiting sparsity

$$\mathcal{D} = \Big\{ (\gamma, \theta, x, \eta) : x_k = -\sum_{i=1}^{na} a_i x_{k-i} + \sum_{j=1}^{nb} b_j u_{k-j};$$
$$m_r m_l x_k = m_l \left(y_k - \eta_k + m_r c_r \right) \chi_{\mathcal{Y}^r}(y_k) + m_r \left(y_k - \eta_k - m_l c_l \right) \chi_{\mathcal{Y}^l}(y_k);$$
$$|\eta_k| \leq \Delta\eta_k, \quad k : y_k \in \mathcal{Y}^r \cup \mathcal{Y}^l \Big\}$$

- $x_k = -\sum_{i=1}^{na} a_i x_{k-i} + \sum_{j=1}^{nb} b_j u_{k-j}$ only depends on the linear system parameters a_i and b_j and on the inner signal samples $x_k, \ldots, x_{k-na}$
- $m_r m_l x_k = m_l \left(y_k - \eta_k + m_r c_r \right) \chi_{\mathcal{Y}^r}(y_k) + m_r \left(y_k - \eta_k - m_l c_l \right) \chi_{\mathcal{Y}^l}(y_k)$ only depends on the backlash parameters m_l, c_l, m_r, c_r and on noise sample η_k
- $|\eta_k| \leq \Delta\eta_k$ only depends on the noise sample η_k

Main properties of the proposed bounding algorithm

Property 1 (Guaranteed relaxed uncertainty intervals)
The true parameter γ_k is guaranteed to lie within the computed interval $PUI_{\gamma_k}^{\delta}$

Property 2 (Monotone convergence to tight uncertainty intervals)
The relaxed interval $PUI_{\gamma_k}^{\delta}$ monotonically converges to the tight interval PUI_{γ_k} as the deep of the relaxation δ increases

Property 3 (Computational complexity)
Identification problems with more than 3000 measurements can be dealt with

Remark: The same properties also hold for $PUI_{\theta_j}^{\delta} = \left[\underline{\theta}_j^{\delta};\ \overline{\theta}_j^{\delta}\right]$

Example

Simulated system

- $\mathcal{B}$: $\gamma^T = [m_r,\ c_r,\ m_l,\ c_l] = [0.247,\ 0.035,\ 0.251,\ 0.069]$
- $\mathcal{L}$: second order model with parameters $[a_1,\ a_2,\ b_1,\ b_2] = [1.7,\ 0.9,\ 2.1,\ 1.5]$.

The **input is a random sequence** uniformly distributed between $[-1,\ +1]$.

Measurements errors

- w_t is corrupted by random additive noise, uniformly distributed between $[-\Delta\eta_t,\ +\Delta\eta_t]$
- error bounds $\Delta\eta_t$ are such that $SNR_w = 15$ db.

Length of measurement data sequence: $N = 2000$

Example

Parameter	$\underline{\gamma}_i^\delta$	True Value	$\overline{\gamma}_i^\delta$	$\Delta\gamma_i$
m_r	0.238	0.247	0.256	0.009
c_r	0.033	0.035	0.036	0.002
m_l	0.239	0.251	0.261	0.010
c_l	0.065	0.069	0.073	0.004

Parameter	$\underline{\theta}_j^\delta$	True Value	$\overline{\theta}_j^\delta$	$\Delta\theta_j$
a_1	1.692	1.700	1.711	0.009
a_2	0.888	0.900	0.912	0.012
b_1	2.035	2.100	2.161	0.063
b_2	1.438	1.500	1.562	0.062

Conclusion

- We presented a one-shot procedure for bounded-error identification of linear systems with backlash nonlinearity
- The proposed approach does not require any constraint on the input signals
- Computation of parameter bounds is formulated in terms of (nonconvex) polynomial optimization
- Guaranteed bounds are computed approximating the global optima by means of suitable (convex) LMI relaxation techniques
- Sparsity structure of the formulated problem is used to significantly reduce the computational complexity of the SDP relaxed problems.
- Convergence to tight bounds is guaranteed